Interference Removal from Electro-Cardiogram Signals

ECE180-01 Lab
Instructor: Dr. Walter
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Written By

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1 Pre-Lab

1.1 Creating FIR Filter with PeZ

With the given FIR system $H(z) = 1 - z^{-1} + z^{-2}$ implement the FIR system using the PeZ demo.

One can simply factor the system to get a function that can be used to find the zeros and poles of the system.

$$\text{H(z) = 1 - z}^{-1} + z^{-2}$$
$$\text{H(z) = z}^{2} - z + 1_{\overline{z^2}}$$

The poles are found from the denominator of the fraction and the zeros are found from the numerator. In this case one can find 2 zeros as seen below.

$$\tfrac{1}{2} + \tfrac{j}{2}\sqrt{3}, \ \tfrac{1}{2} - \tfrac{j}{2}\sqrt{3}$$

Using these zeros one can place the 2 zeros using the PeZDemo Tool as seen below.

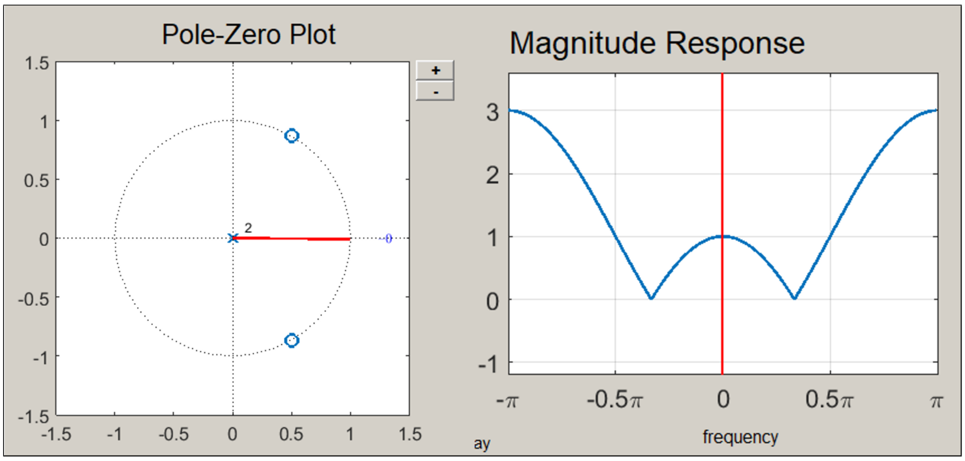

Figure 1: Pole-Zero and Magnitude Response

1.2 FIR Filter with Many Zeros

$$H(z) = \frac{1-z^{-6}}{1-z^{-1}} \tag{1}$$

To determine the zeros and poles one must once again find the zeros of the numerator and denominator.

Numerator Zeros

$$1, -1, \frac{\sqrt{-2-2j\sqrt{3}}}{2}, -\frac{\sqrt{-2-2j\sqrt{3}}}{2}, \frac{\sqrt{-2+2j\sqrt{3}}}{2}, -\frac{\sqrt{-2+2j\sqrt{3}}}{2}$$

Denominator Zeros (Poles)

$$1$$

In this case this function involves a numerator-denominator cancellation that removes the zero at z=1 resulting in the following roots of the function overall.

Zeros

$$-1, \frac{\sqrt{-2-2j\sqrt{3}}}{2}, -\frac{\sqrt{-2-2j\sqrt{3}}}{2}, \frac{\sqrt{-2+2j\sqrt{3}}}{2}, -\frac{\sqrt{-2+2j\sqrt{3}}}{2}$$

Placing these points on the PeZ will give one the following graphs.

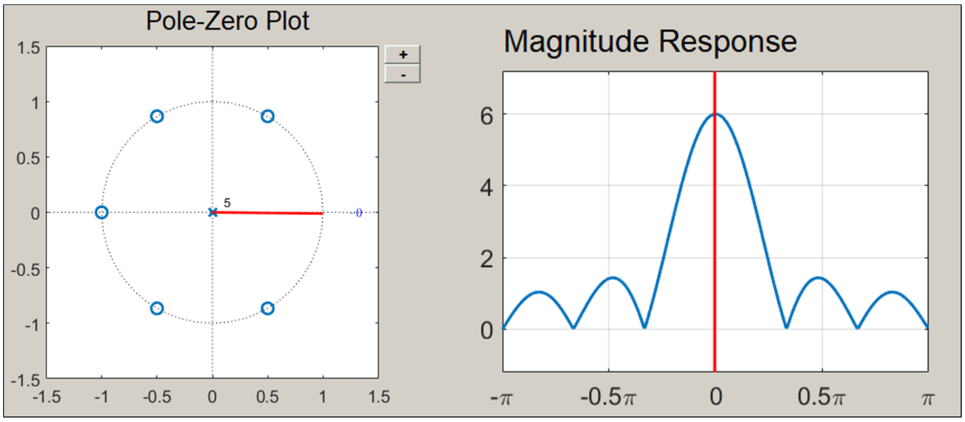

Figure 2: Pole-Zero and Magnitude Response

One can then look at the Magnitude Response graph to find the various points where the filter nulls the frequency.

List of Nulled Frequencies

$$\frac{\pi}{3}, \frac{\pi}{1.5}, 1, \frac{\pi}{-1.5}, \frac{\pi}{-3}, 1$$

1.2.1 Filter with Six Zeros

Using the numerator of the previous filter as the entire system function yields these results. One can see the plots of the filter below.

$$H(z) = 1 - z^{-6}$$

Zeros

$$1, -1, \frac{\sqrt{-2-2i\sqrt{3}}}{2}, -\frac{\sqrt{-2-2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}, -\frac{\sqrt{-2+2i\sqrt{3}}}{2}$$

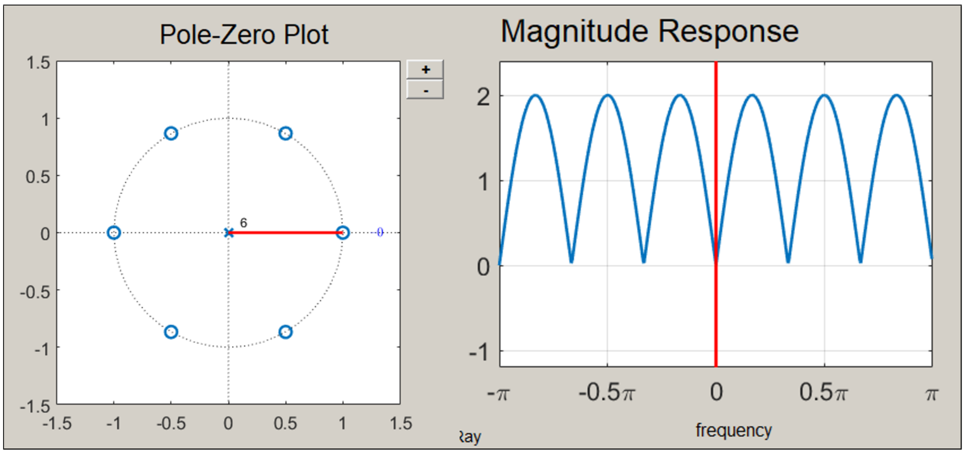

Figure 3: Pole-Zero and Magnitude Response

One can see that this is a comb filter by looking at the shape of the magnitude response graph and how it resembles a comb.

List of Nulled Frequencies

$$0, \frac{\pi}{3}, \frac{\pi}{1.5}, -\pi, \frac{\pi}{-1.5}, \frac{\pi}{-3}, \pi$$

Moving the zero that was -1 to 1 yields this system function

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - 2z^{-3} + 2z^{-4} - 2z^{-5} + 1z^{-6} \tag{2}$$

and creates the following graphs.

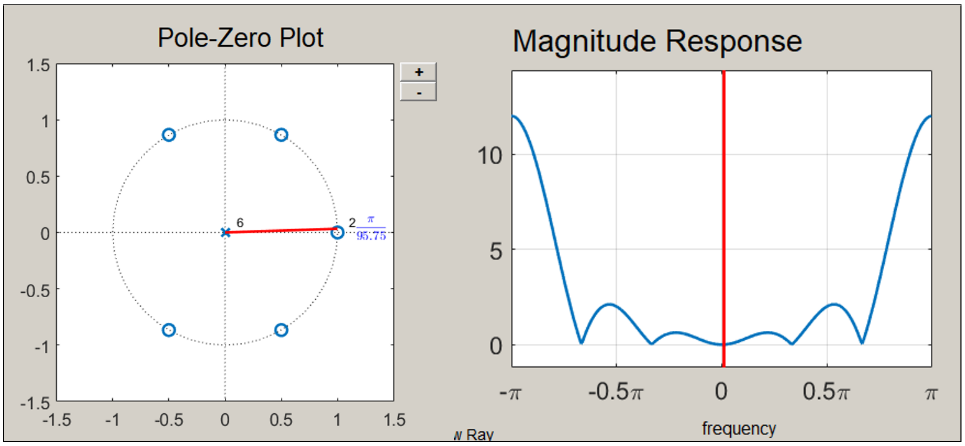

Figure 4: Pole-Zero and Magnitude Response

One can see that this is a high pass filter based on the magnitude response graph and the location of the zeros in the pole-zero plot.

1.3 Real Poles

Placing zeros at z = 1 and placing a pole at $z = \frac{-1}{2}$ yields the following graphs and equation.

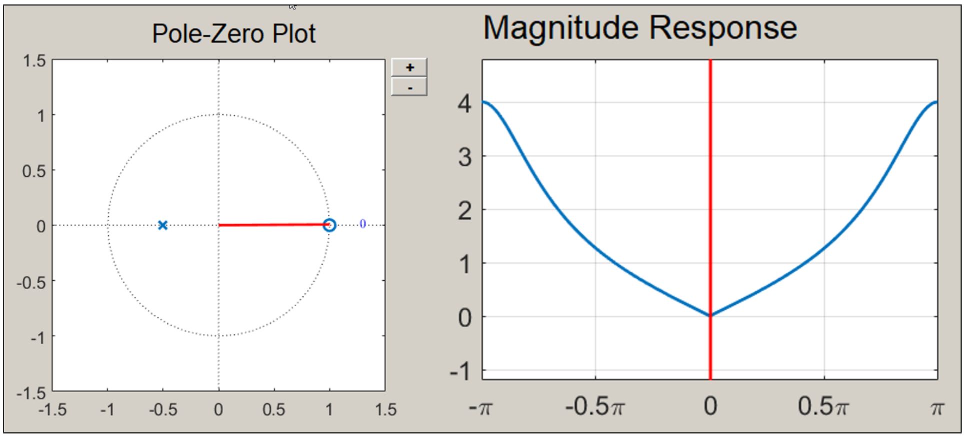

Figure 5: Pole-Zero and Magnitude Response

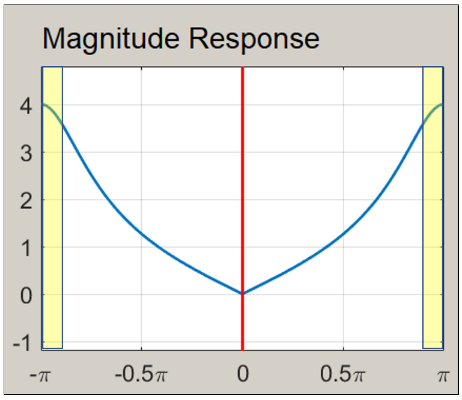

Figure 6: Pass Band Regions

When moving the pole the frequency response becomes sharper in way. This continues until the pole is then moved outside of the unit circle. When this happens the filter becomes unstable. One can see the magnitude response "blow up" as the scale of the chart increases. To guarantee system stability one must place the zeros inside of the unit circle so the output doesn't blow up.

Movement Video Link

$$H(z) = \frac{1 - z^{-1}}{1 + 0.5z^{-1}} \tag{3}$$

1.4 Creating a First-Order IIR Filter with PeZ

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}} \tag{4}$$

One will find a Zero at 1 and a Pole at -0.9.

One can see below that moving the pole from -0.9 which is a high pass filter changed the range of frequencies that are passed and turns it into a filter which rejects frequencies that are zero in this case.

Movement Video Link

1.5 Creating a Second-Order IIR BPF with PeZ

$$H(z) = \frac{1 - z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}} \tag{5}$$

Factoring the system and solving for the poles and zeros gives these values

Zeros

1,-1

Poles

-0.4000000000 + 0.6928203230j, -0.4000000000 - 0.6928203230j

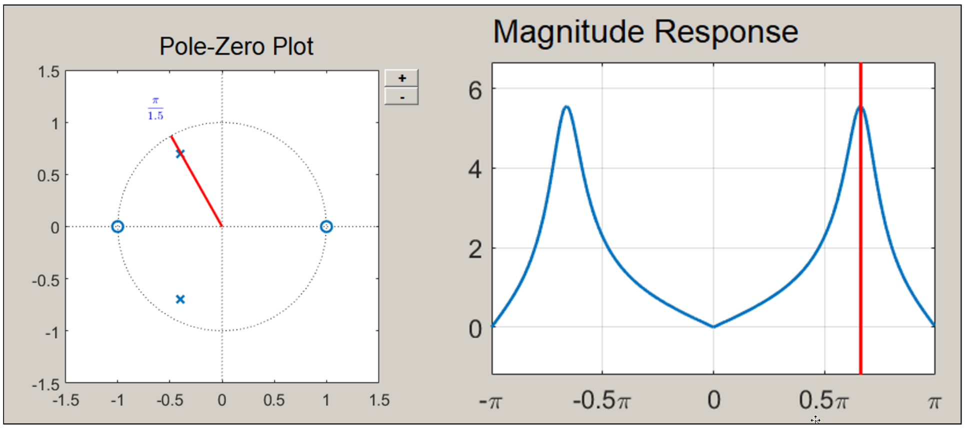

Figure 7: Pole-Zero and Magnitude Response

One can clearly see by the magnitude response that this is a band pass filter.

1.6 Using Poles to Create Filters that are Not Bandpass Filters

$$H(z) = \frac{64 + 80z^{-1} + 100z^{-2}}{1 + 0.8z^{-1} + 0.664z^{-2}} \tag{6}$$

Zeros

$$-\frac{5}{8}, \frac{5i}{8}\sqrt{3}, -\frac{5}{8} - \frac{5i}{8}\sqrt{3}$$

Poles

-0.4000000000 + 0.6928203230j, -0.4000000000 - 0.6928203230j

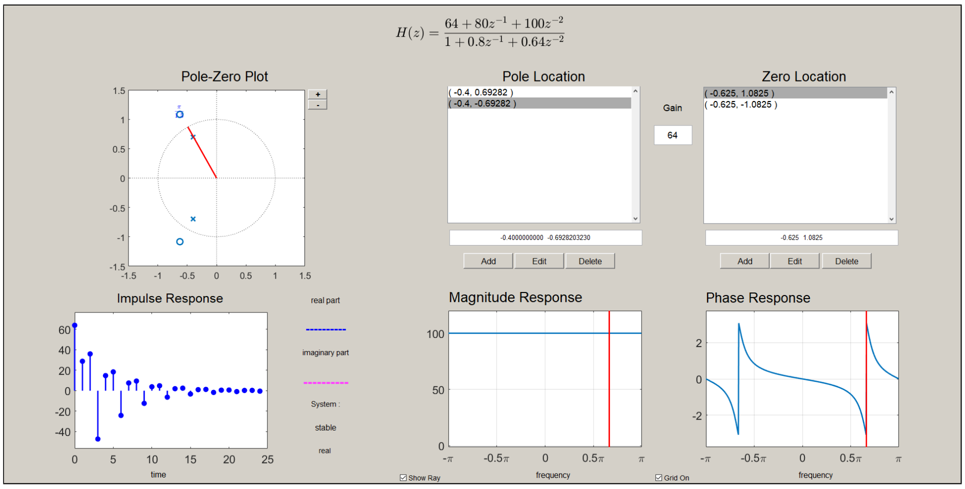

Figure 8: PeZ Screen

The gain was set to 64 for the coefficients to be correct in the numerator.

Looking at the magnitude Response one can see that this filter simply passes through values. Moving the zeros to the unit circle allows you to null those certain frequencies. In this case it was $\pm\frac{\pi}{1.5}$. Below is a link to a video showing this movement.

Video Link

2 Lab Exercise

2.1 IIR Allpass Filter

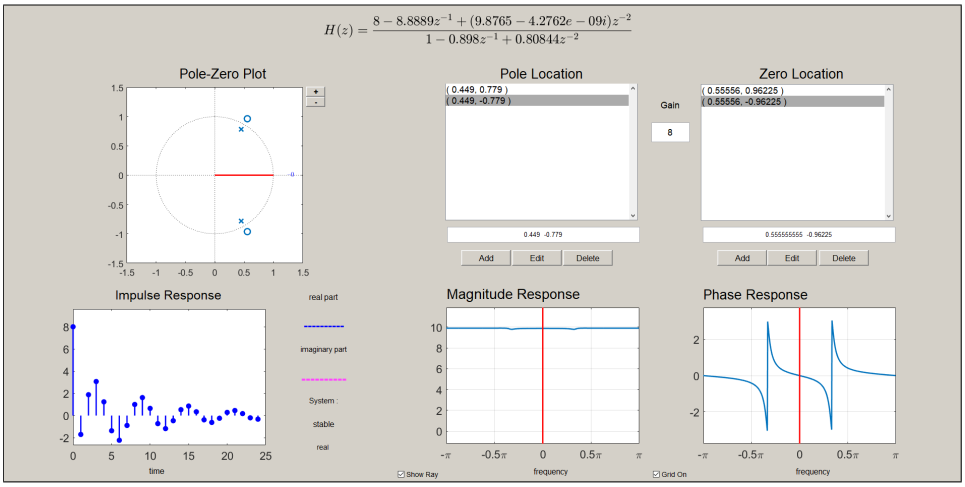

Figure 9: All Pass Filter PeZ Demo Screen

$$H(z) = \frac{8 - 8.8889z^{-1} + (9.8765 - 4.2762e - 9j)z^{-2}}{1 - 0.898z^{-1} + 0.80844z^{-2}} \tag{7}$$

2.2 IIR Notch Filters

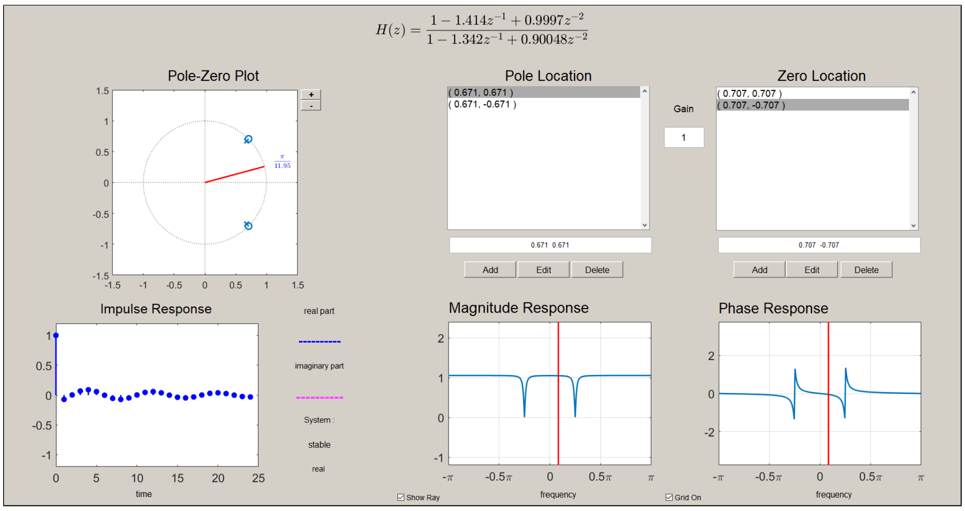

Figure 10: IIR Notch Filter

$$\frac{1 - 1.414z^{-1} + 0.9997z^{-2}}{1 - 1.342z^{-1} + 0.90048z^{-2}} \tag{8}$$

2.3 IIR Filter Implementation

```
%% Section 2.3

n = 1:51;
hh = filter([1,2],[1,-0.9],[1,zeros(1,50)])
stem(n,hh);
```

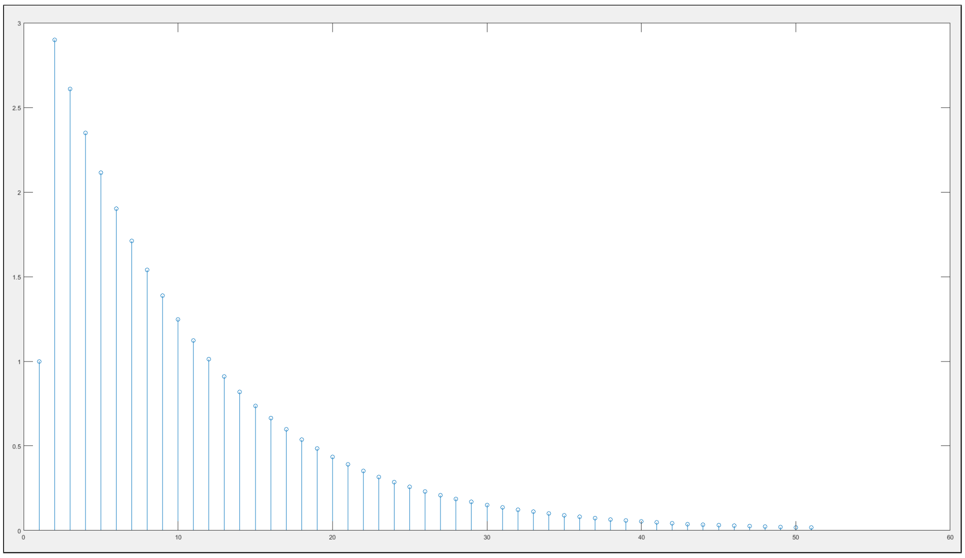

Figure 11: Stem Plot

2.4 Notch Filter Removes Sinusoidal Interference

2.4.1 Part A

```
% fs = sampling frequency in Hz
fs = 8000;

% fint = frequency of the interfering sinusoid (near 50 or 60 Hz)
fint = 60;

% IDstring = your login ID as a string, e.g.,     gburdell7     or
      gtg555q
IDstring = 'gburdell7'

% dur = duration of the signal (optional); default = 15 secs.
dur = 9;

SIG = ECGmake(IDstring, dur);

% Plotting the signal
plot([1:length(SIG)],SIG)
```

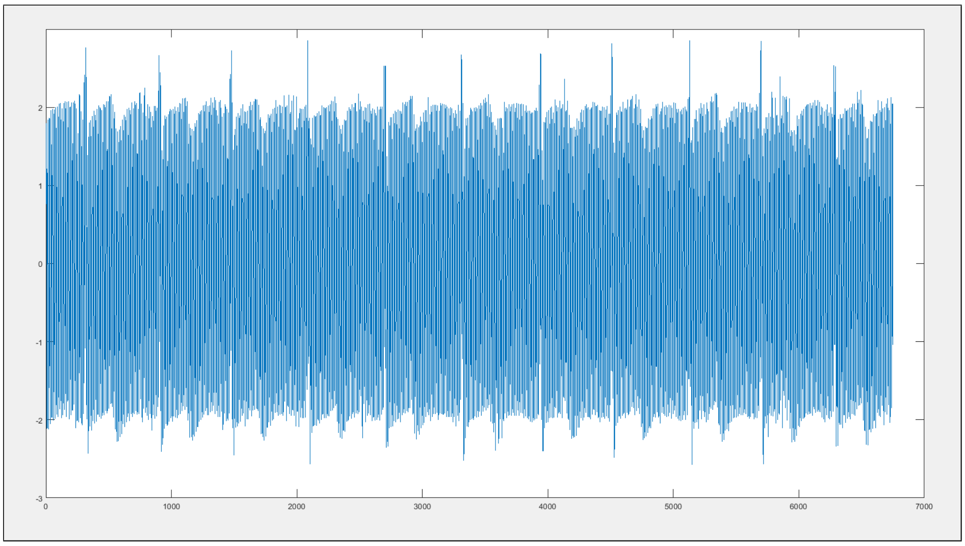

Figure 12: ECG Make Full Signal No Filter Applied

ECG Make was ran for 9 seconds with a **60Hz interference** generating 6750 points making a **sampling rate of 750 Hz**.

2.4.2 Part B

One can use the formula below to determine the frequency to remove the interference.

$$\frac{fint}{\frac{fs}{2}} \tag{9}$$

In this case the frequency that should be nulled is 0.16 π. Using this one can easily determine the pole and zero placement for the previously used notch filter.

Zeros: $e^{\pm j\theta}$ **Poles:** $re^{\pm j\theta}$

In this case r = 0.9 was used and that created the following pole zero diagram and magnitude response.

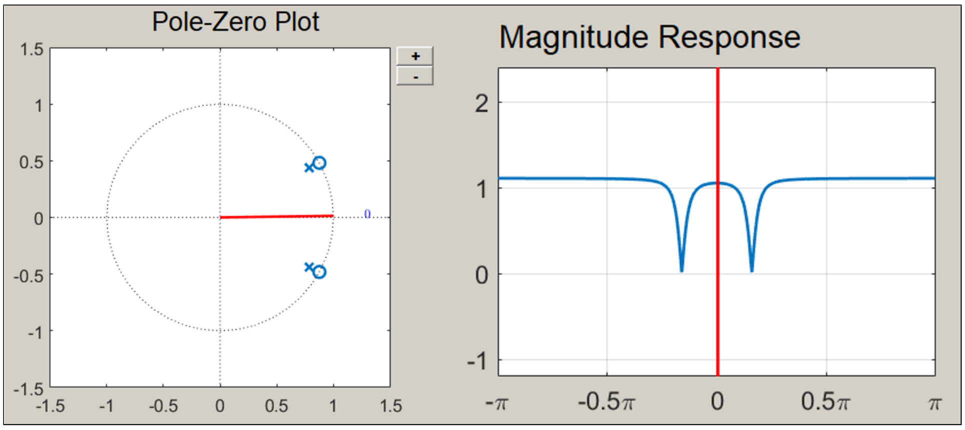

Figure 13: Pole-Zero Plot and Magnitude Response

2.4.3 Part D

$$H(z) = \frac{1 - 1.752z^{-1} + 0.99874z^{-2}}{1 - 1.5772z^{-1} + 0.81172z^{-2}} \tag{10}$$

```
% This code shows the filter coefficients being applied to a filter
% that takes in the signal and then proceeds to plot it
plot(filter([1,-1.752,0.99874],[1,-1.5772,0.81172],SIG))
```

2.4.4 Part E

```
%% Sub Plots
figure;
subplot(1,2,1)
```

```
plot([1:length(SIG)],SIG)
subplot(1,2,2)
plot(filter([1,-1.752,0.99874],[1,-1.5772,0.81172],SIG))
```

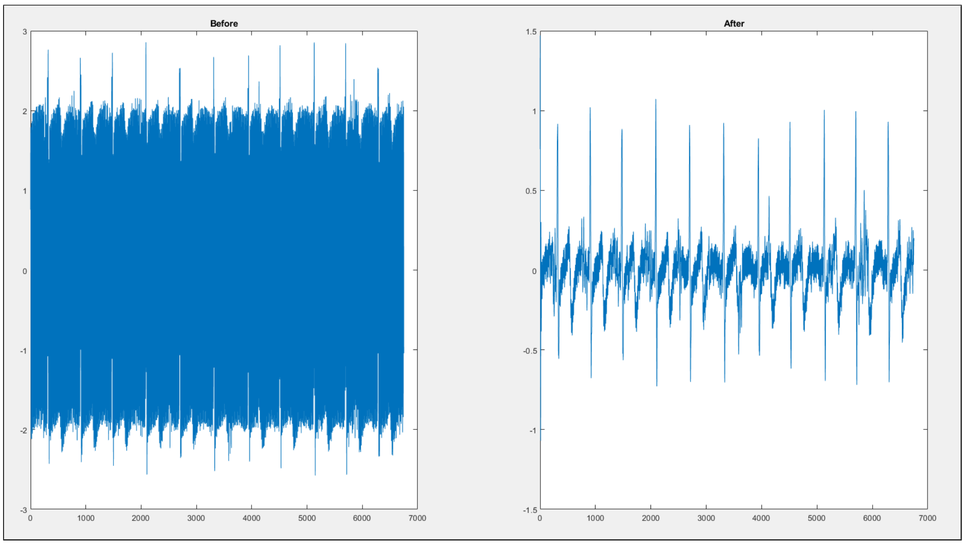

Figure 14: Full Signal Comparison

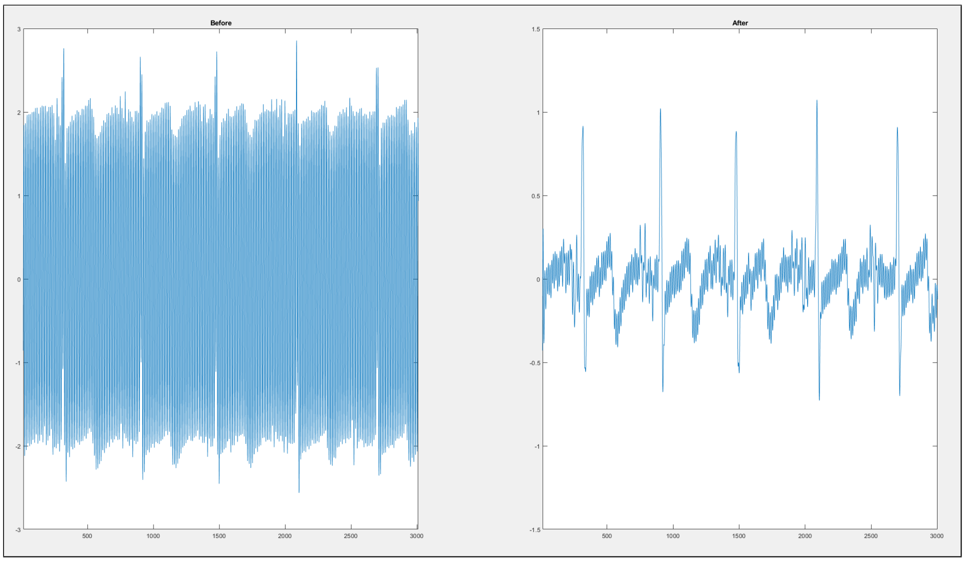

Figure 15: Zoomed In signal